

Marking Starting Locations for the 400-Meter Dash Track and Field Event

I. ASSESSMENT TASK OVERVIEW & PURPOSE:

The students will apply their knowledge of circle properties and the law of cosines to determine the proper placement of the lanes of an Olympic regulation track using specifications published by the International Association of Athletics Federation (IAAF).

II. UNIT AUTHORS:

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III. COURSE:

Geometry and Trigonometry

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

The learner will be able to solve real-world problems associated with circles using properties of arcs.

VI. REFERENCE/RESOURCE MATERIALS:

Compass, Measuring Tape or Measuring Wheel. Computers will be used to research information concerning Olympic regulations.

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes two assessment components. First, students will assess themselves using a checklist. The teacher will use the same checklist to evaluate the student's analysis and understanding of circle properties and the Law of Cosines.

The assessment also analyzes a student's ability to check for reasonableness by providing an alternative method to support their conclusions and considering real-life contexts with regard to the solution.

VIII. EVALUATION CRITERIA:

The rubric provides a checklist for students and teachers in evaluating the work. Actual student work will be available as benchmarks and examples of product expectations.

IX. INSTRUCTIONAL TIME:

Two ninety-minute class periods or four forty-five minute class periods.

Marking Starting Locations for the 400-Meter Dash

Strand: Geometry

Mathematical Goals and Objective(s):

This assessment provides a real life activity in which students gain experience applying circle properties and will discover the Law of Cosines.

Related SOL:

G.11 The student will use angles, arcs, chords, tangents, and secants to

- a) investigate, verify, and apply properties of circles;
- b) solve real-world problems involving properties of circles; and
- c) find arc lengths and areas of sectors in circles.

T.9 The student will identify, create, and solve real-world problems involving triangles. Techniques will use the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

NCTM Standards

Geometry Content Standards:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Process Standards:

- Apply and adapt a variety of appropriate strategies to solve problems.
- Build new mathematical knowledge through problem solving.
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others.

Materials/Resources

- Compass, Measuring Tape or Measuring Wheel. Computers will be used to research information concerning Olympic regulations.

Assumption of Prior Knowledge

- In order for students to be successful at this task, they should have received instruction in solving real world problems involving circles, most specifically involving arc lengths.
- In order to be successful at this task a student should be Analysis level on the Van Hiele scale with respect to understanding the significance of circle properties and arc length concepts.

Introduction: Setting up the Mathematical Task

- Students should be placed in groups of four for this task, so that each student has a partner to work with and another pair to share during table group events.

- To get students started thinking about this task you begin with asking how many students watch the Olympics or have participated in a race of some kind. Even those who have never competed in Track and Field are generally familiar with the track on which races are held. With the arrival of spring every year, comes the annual competitions in the world of Track and Field. In every level of competition, it is not unusual to see races decided by fractions of seconds. Perhaps the most important aspect of Track and Field is that everyone must compete under the exact same circumstances. The most basic form of consistency is to ensure that all competitors run the **EXACT** same distance. Because races can be won or lost by tenths or even hundredths of seconds, any variations in the actual distance to be run among competitors can easily be the difference between “bringing home the gold” and going home empty-handed.
- Ask them to discuss the design of the track (two semicircles with one straight length on each side). The following are questions you might ask students to think about before the activity begins. Do all race participants finish the race in the same location? Why is that necessary? Does that seem fair given the construction of the track? How about the starting locations? Have they noticed how runners starting positions are staggered? What could be a possible reason for the different locations for starting?
- Teacher will distribute materials and allow students to read over directions and look at rubric. The teacher will provide an explanation of the overall project including parts to be completed individually and those that may be done with a partner. The teacher will review the rubric and provide student work examples.
- Teacher will monitor student progress and provide assistance as needed as students complete the task.
- Access to computers and the internet should be provided throughout the task to support students' research of the problem at hand.

Student Exploration

Students will be encouraged to use notes, textbooks, and technology to clarify aspects of the task that confuse them. As students begin to work, the teacher will encourage groups to identify the important information that they will need in order to complete this task, determine the best methods for finding this information, and divide the workload to do so. The teacher will also encourage students to reflect on prior experiences in the class and reflect upon which skills discussed in class may be needed to complete the task. The teacher should anticipate hearing discussions of concepts like circumference, perimeter, radius, arc lengths.

Student/Teacher Actions

Students will be considering the measurements of their school's track. Some considerations might include: How can the center of the semicircle be located? Is the curved section of the track circular or elliptical? Does it matter? What margin of error is acceptable?

The teacher will monitor student progress and provide assistance, as necessary. Students might begin to express the notion that the imprecision in measurement would not hold up under scrutiny.

It might be difficult for students to develop The Law of Cosines or discover the use of a theodolite. An attachment, Monitoring Student Responses, is provided to highlight student responses that might be included.

Evaluation Rubric/Benchmarks:

Students will submit a copy of the following rubric with their completed project. They will complete the checklist and provide justifying comments. Teachers will complete the same rubric providing a score in each element area that applies to the student's work.

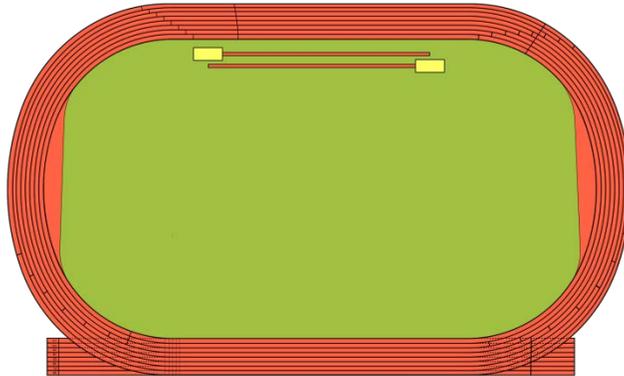
Benchmark: See Student Work Samples (High, Medium, Low)

Names: _____

Period: _____

TRACK AND FIELD PERFORMANCE TASK

According to specifications published by the International Association of Athletics Federation (IAAF), the standard track for Track and Field races is an oval with a total distance of 400 meters. The track has two straight distances of 84.39 meters each and two semi-circular distances of 115.611 meters each (for lane 1). A track will also have at least eight lanes with a width of 1.22 meters (your track for this project may be different).



The Problem: Questions about distance are irrelevant when a race is conducted from start to finish on a straight section such as the 100-meter dash (this race uses an extension of the straight sections to avoid any turns). However, when the race is longer and includes one or both turns, then questions of distance are extremely relevant because competitors are required to stay in their own lanes for races such as the 200-meter and 400-meter dash. The focus for this project will be the 400-meter dash. Since most tracks already have the staggered starting lines for different lanes marked, your task is to verify that your school's track was marked correctly. Each group will be given a different lane to verify. Note: In order to ensure the finish line is at the end of a straight distance, the start line for lane 1 is at the start of a turn.

Task 1: Using the skills and concepts you have learned this year, you are to take whatever measurements are necessary, for your school's track, to calculate how far the starting line for your assigned lane should be advanced. Included in your solution should be sketches, indications where measurements were taken, and completed formulas showing all Geometry and Algebra work.

Task 2: Having completed task 1, you must develop and implement a process for marking your assigned lane with the proper start line. Your solution must include a detailed description of all steps required to mark the starting line for your assigned lane properly.

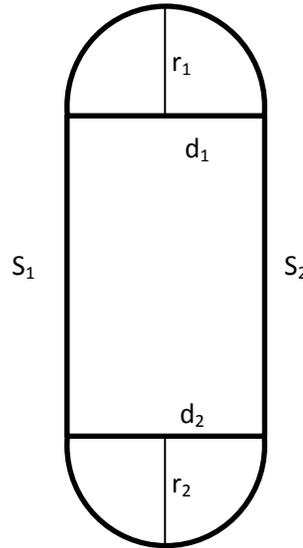
Reflection: Critically evaluate your solutions for task 1 and task 2. Would your solution hold up under international scrutiny? If not, identify the source of any potential inaccuracies. Additionally, consider and describe in detail any alternative methods that could have been used.

Monitoring Student Responses – Track and Field Performance Task

Task 1:

Real world measurements taken

1. Measure straight track S_1 and S_2 .
2. Measure diameter of semicircles d_1 and d_2 .
3. Find the center of semicircles $\frac{d_1}{2}$ and $\frac{d_2}{2}$
4. Measure radius of each semicircle r_1 and r_2 .
5. Measure width of lane.



Record all measurements:

$$S_1 = \underline{84.39 \text{ m.}}$$

$$S_2 = \underline{84.39 \text{ m.}}$$

$$d_1 = \underline{73.6 \text{ m}}$$

$$d_2 = \underline{73.6 \text{ m.}}$$

$$r_1 = \underline{36.8 \text{ m.}}$$

$$r_2 = \underline{36.8 \text{ m.}}$$

$$w = \underline{1.22 \text{ m.}}$$

Answers will vary from track to track. Students should verify total length of lane 1 and that turns are truly semicircular. These responses are responses for students assigned lane 2.

Answer Key

Task 1:

- Total distance of curves

$$2(115.611) \text{ of } 231.222$$

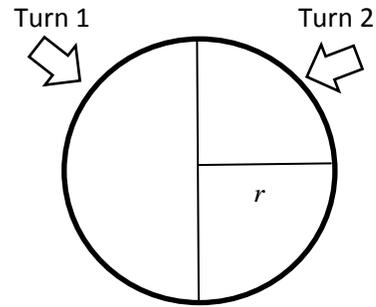
- Find radius of curve

$$C = 2\pi r$$

$$231.222 = 2\pi r$$

$$r = \frac{231.222}{2\pi}$$

$$r = 36.800 \text{ (rounded to thousandths)}$$



- Find the circumference of lane 2

$$r + 1.22 = 38.02$$

$$C_{\text{lane2}} = 2\pi r$$

$$= 2\pi(38.02)$$

$$= 238.887 \text{ (rounded to thousandths)}$$

- Find Δ distance between lane 1 and lane 2

$$238.887 - 231.222 = 7.665 \text{ m}$$

Lane 2's start line should be advanced 7.665 meters.

Answer Key

Task 2:

Method 1

- Use a meter wheel and measure out 7.665 m along the inside lane marker for lane 2.
- 0665 m can be measured by rolling the meter wheel on a meter stick, then marking the wheel when it has covered 0.665 meters.
- Once 7.665 m is measured out along the inside of lane 2, the start line is marked across lane 2 along a radius of the semicircle that goes through the point marked.

Task 2:

Method 2

- Find the central angle that produces an arc length of 7.665 m

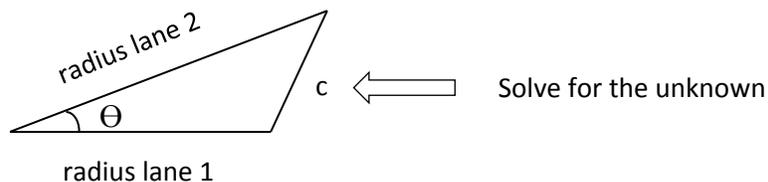
$$\text{arc length} = C \cdot \frac{\theta}{360}$$

$$7.665 = 238.887 \left(\frac{\theta}{360} \right)$$

$$\theta = \frac{7.665 (360)}{238.887}$$

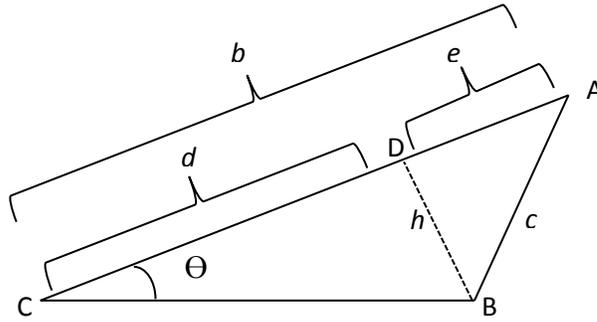
$$\theta = 11.551^\circ$$

- Find the straight line distance from the inside edge of lane 1 start line to inside edge of lane 2 start line.



Answer Key

$a = \text{radius lane 1 (36.8 m)}$
 $b = \text{radius lane 2 (38.02 m)}$
 $c = \text{unknown}$
 $\theta = 11.551^\circ$
 $d + e = b$
 $h = \text{altitude of } \triangle ABC$



For $\triangle ABC$:

$$\sin \theta = \frac{h}{a} \quad h = a \sin \theta$$

$$\cos \theta = \frac{d}{a} \quad d = a \cos \theta$$

$$e = b - d \quad e = b - a \cos \theta$$

For $\triangle DAB$:

$$c^2 = h^2 + e^2$$

$$c^2 = (a \sin \theta)^2 + (b - a \cos \theta)^2$$

$$c^2 = a^2 \sin^2 \theta + b^2 - 2ab \cos \theta + a^2 \cos^2 \theta$$

$$c^2 = a^2 \sin^2 \theta + a^2 \cos^2 \theta + b^2 - 2ab \cos \theta$$

$$c^2 = a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 - 2ab \cos \theta$$

$$c^2 = a^2 (1) + b^2 - 2ab \cos \theta$$

$$c^2 = (36.8)^2 + (38.02)^2 - 2(36.8)(38.02) \cos (11.551)$$

$$c = 7.626 \text{ m}$$

- Use tape measure to mark distance c from point b to point a . Then extend start line for lane 2 along a radius that goes through point a .

Answer Key

Task 2:

Method 3 (this method can be discovered with internet searches)

- Find the central angle that produces an arc length of 7.665 m
** see solution in method 2 **
- Using a theodolite at the center of the semicircle, have someone with a marking stake with level, move along the inside of lane 2 until the correct central angle is made. Mark the starting line for lane 2.
- Extend the starting line across lane 2 along a radius that goes through the marked point.

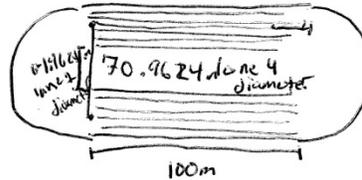
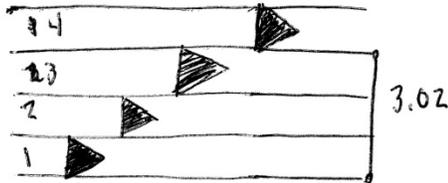
SCORING RUBRIC for Track and Field Performance Task

TASK 1				
Measuring	Nothing to show that measurements were taken.	Not all required measurements were taken.	All required measurements were taken, but measuring process lacked detailed explanation.	All required measurements were taken with detailed step-by-step explanation of process.
Self-assessment	0	1 2	3 4	5 6
Teacher assessment	0	1 2	3 4	5 6
Diagram/sketch	No diagram	Incomplete diagram. Doesn't show all the measurements that were taken.	Diagram shows all measurements, but lacks detail or data recorded.	All measurements taken were diagrammed and recorded in detail.
Self-assessment	0	1	2	3
Teacher assessment	0	1	2	3
Calculating	No explanation or formulas used.	Incorrect formula or inaccurate explanation given for start line adjustment distance.	Correct formula used, but incorrect/no explanation, or algebraic work not shown.	Correct formulas used, complete explanation provided, and algebraic work shown.
Self-assessment	0	1 2 3	4 5 6	7 8 9
Teacher assessment	0	1 2 3	4 5 6	7 8 9
TASK 2				
Marking the start line	No explanation of method.	Explanation of method provided, but process had serious flaws.	Logical process used, but explanation was confusing or unclear.	Logical process used with clear and concise explanation of method.
Self-assessment	0	1 2	3 4	5 6
Teacher assessment	0	1 2	3 4	5 6
Accuracy evaluation	No evaluation of accuracy given.	One or more potential sources of inaccuracy were omitted, or explanations were vague.	Most sources of potential inaccuracy were identified and clearly explained.	All potential sources of inaccuracy were identified and explained.
Self-assessment	0	1 2	3 4	5 6
Teacher assessment	0	1 2	3 4	5 6
Alternate method	No alternate method given.	Alternate method was provided but it would not work.	Workable alternate method provided but explanation was incomplete.	Workable alternate method provided with clear and complete explanation.
Self-assessment	0	1 2	3 4	5 6
Teacher assessment	0	1 2	3 4	5 6
OVERALL				
Diagrams	No diagrams included.	No diagrams included.	Diagrams were incomplete, vague, or unclear.	Diagrams were complete, precise, and clear.
Self-assessment	0	0	1	2
Teacher assessment	0	0	1	2
Explanations	No explanations included.	No explanations included.	Explanations were incomplete, vague, or unclear.	Explanations were complete, precise, and clear.
Self-assessment	0	0	1	2
Teacher assessment	0	0	1	2
TOTAL SCORE (out of 40 points):		SELF ASSESSMENT: _____		TEACHER ASSESSMENT: _____

STUDENT WORK SAMPLE - HIGH

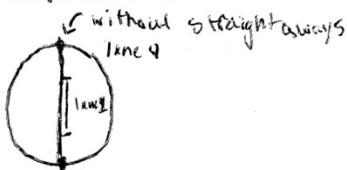
Track Problem

First, we measured the distance between the start and end of the curve; that was 70.9624 m from lane 3 to lane 4 and 64.9624 m from lane 1 to lane 1. Then, we measured from the inside of lane 1 to the inside of lane 4; that was 3.02 . Also, the straight aways are 100 m



~~Method one~~

- ① $(\pi d + 200) - (\pi d + 200) = \text{offset}$
- ② $(\pi 70.9624 + 200) - (\pi 64.9624 + 200) = 0$
- ③ $(422.4319545) - (404.0353986) = 0$
- ④ $18.39655592 = 0$



Method one - Meter wheel

The first method is to walk the lanes using a meter wheel.

con - You can't get a far decimal using the meter wheel

pro - not much more math is needed

what I did is I found the circumference with both of the diameters. Then I subtracted the larger one (lane 4) from the smaller one (lane 1).

The offset is

~~18.39655592~~

18.4316531

Method two - rope

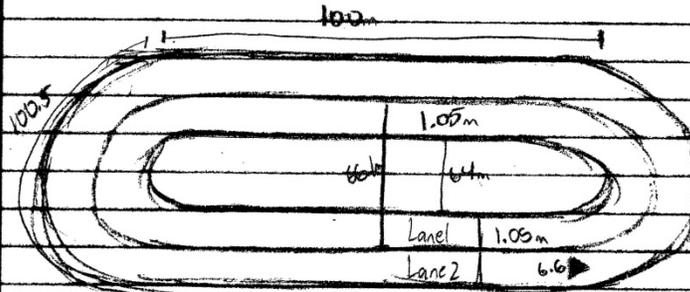
Measure a rope that is tight and straight. Then bend the rope along the line.

con - Super inaccurate with bends in the rope

pro - you could be a lot more accurate

* Measurements are as close as possible to the actual lengths

STUDENT WORK SAMPLE – MEDIUM



6-14-11

$$C_1 = 64\pi + 1.05\pi + 1.05$$

$$66.15\pi$$

$$64\pi$$

$$66.15\pi - 64\pi = 6.597344573\pi$$

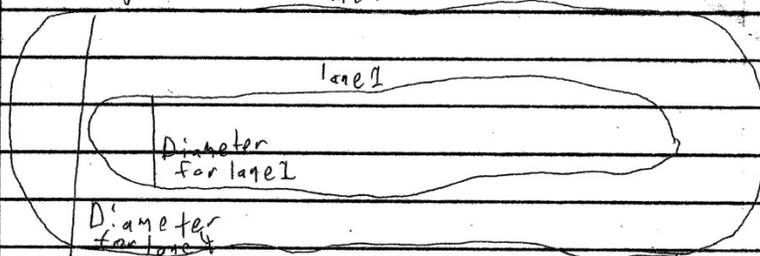
$$207.6592744m - 201.0619298m$$

$$207.6592744 - 201.0619298 = 6.5973446m$$

Every track is 400m however, as it extends out each starting line changes. Since in our school the track is not verified, it has different measurements and starting lines. For my project we had to find where lane 2 starts, and my answer would be 6.6 meters. I found the answer by measuring their diameters which are 66m and 64m. By using those, I found both of the circumferences and since lane 2 is longer than lane 1, I did $C_2 - C_1$ or the circumference of lane 2 minus the circumference of lane 1 to get the distance where the starting of lane 2 should be. Another way I wanted to do it with was using arc lengths. Anyway, my final answer is 6.6.

STUDENT WORK SAMPLE - LOW

Finding the offset lane 4



Lane 1 Length: πd : $\pi 64.9624 = 203.9597349$

Lane 4 Length: πd : $\pi 70.9624 = 222.9349545$

Stagger: lane 1 length - lane 4 length = 18.9752196

Implementing the offset

In order to offset the starting line we take the semi-circles and make them into a full circle. Then we find the circumference of the circle. We use the offset to find out the percent of the circle it is. Use the percent to find the angle out of 360° and use a protractor to put a rope onto the semi-circle, therefore finding the offset.

Inaccuracies

We may be inaccurate when we use the protractor because in order for it to be the right angle the rope must be taut. Also we may have had measurement inaccuracies for the same reason, the rope or the measurer not being taut. I also made calculation errors.